Tanta University

Department of Electronics and Electrical Communication Engineering

Faculty of Engineering

Course: **Electromagnetic Waves (1)**
Date: **Thu., 26-June-2014** ***(Second term)***,

Course Code: **EEC2208**,
Time Allowed: **Three hours**,

Students: **2nd year**
No. of Pages: **2**,

Final Exam

(Total marks: 85 Marks)

Answer the following questions:

Problem 1: **(15 marks)**

a) State the Maxwell's equations in the point and integral forms.

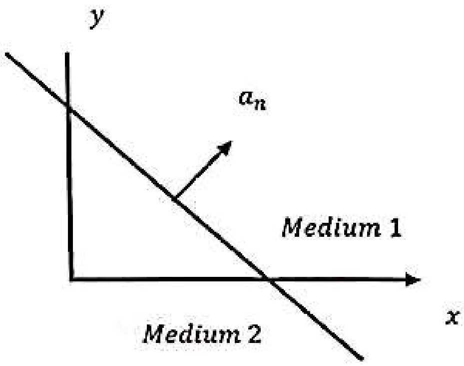
b) Drive the wave equations that represents the field components for a wave traveling through a source free medium.

c) If the magnetic field $H = \frac{E_m}{\eta} e^{j(\omega t - \beta z)}.a_x$. Find all other electric and magnetic field components in a source free medium with permeability μ_r and permittivityϵ_r. Find the direction of propagation for that wave.

Problem 2: **(15 marks)**

a) Drive the relation between tangential and normal components of the electric and magnetic fields at the boundary between two dielectric media.

b) Two dielectric media, which are interfaced as shown in the figure. If the magnetic field in the first medium is $H_1 = 0.3a_x - a_y + 0.3a_z$ and the electric field in the second medium is $E_2 = 1.2a_x + 7.5a_y$. Find all the fields in the two media if $\sigma_1 = \sigma_2 = 0, \epsilon_{r_1} = 10, \epsilon_{r_2} = 30, \mu_{r_1} = 0.5\mu_{r_2} = 2$.



Problem 3: **(20 marks)**

a) Obtain the parameters: α, β, η, skin depth (δ), the phase velocity (v_{ph}), and group velocity (v_g) for a wave propagating in a good dielectric medium then, write down the wave equations for each case.

b) An electric field of a plane wave with the value $E(y,t) = 5 \times 10^{-5} e^{-\alpha y}.\cos(0.2\pi \times 10^9 t + \beta y)$ is propagating in a medium with $\sigma = 5 \times 10^{-6}\ \Omega^{-1}/m$, $\epsilon_r = 300$.

i. Determine if the medium where the wave propagates is a good dielectric or good conductor.

ii. Find the range of frequencies that change the behavior that is obtained in (i).

iii. Evaluate $\alpha, \beta, \eta, \delta, v_g, v_{ph}$ and then find H.

iv. Find the Poynting vector in this medium.

Problem 4: (15 marks)

a) What is meant by the wave polarization? What are its types? Drive the equations relating the magnitude of electric field components for each type.

b) An electric field is propagating in a medium that is found to be,

$$E(x,t) = 4\times10^{-3}e^{-6x}\cos(5\pi\times10^{9} - 10\pi x)\,a_y + E_o e^{-\alpha x}\cos(\omega t - kx + \theta_o)\,a_z$$

i. Find the type of the medium and σ, ϵ_r. Consider $\mu_r = 1$.

ii. Find E_o, θ_o if:

1. The wave is linearly polarized and inclined by 40^o on the y-axis
2. The wave is circularly polarized
3. The wave is elliptically polarized

Problem 5: (20 marks)

a) Drive expressions for the reflection and transmission coefficients for a plane wave that is traveling through an interface between two dielectric materials (consider $\sigma = 0$ for the two dielectrics). Consider that the incident wave is linear perpendicular polarized.

b) A linear perpendicular polarized wave travels through two dielectric media ($\sigma = 0, \mu_r = 1$ for each). They are interfaced at the $x - z$ plane. The first dielectric occupies the $+ve\ y - direction$, while the second dielectric occupies the $-ve\ y - direction$. The plane-wave travels from the first to the second dielectric. If the incident electric field:

$$\vec{E} - 10^{-6}\times e^{j(6\pi\times10^{8}t - 4\pi x + 9\pi y)}a_z\ V/m$$

And the relative permittivity of the second medium is 300, *Find:*

i. β, η for each medium
ii. The incidence, reflection and transmission angles.
iii. The reflection coefficient, R, and transmission coefficient, T.
iv. The electric and magnetic fields for the incident, reflected and transmitted waves.
v. The critical incidence angle that cause total reflection.

Constants:

Permeability of free space, $u_o = 4\pi\times10^{-7} H/m$,

Permittivity of free space, $\epsilon_o = \frac{1}{36\pi}\times10^{-9}\ F/m.$

With best wishes of success

Dr. Sameh A. Napoleon

Q1 a: Maxwell's eqn's

point form

$\bar{\nabla}\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$

$\bar{\nabla}\times\bar{H} = \sigma\bar{E} + \frac{\partial\bar{D}}{\partial t}$

$\bar{\nabla}\cdot\bar{D} = \rho_v$

$\nabla\cdot\bar{B} = 0$

integral form

$\int \bar{E}\cdot\bar{dl} = -\frac{\partial}{\partial t}\int_s \bar{B}\cdot\bar{ds}$

$\int H\cdot dl = I_{ert} + \frac{\partial}{\partial t}\int_s \bar{D}\cdot\bar{ds}$

$\oint_s \bar{D}\cdot\bar{ds} = \int_v \rho_v\, dv = Q$

$\oint_s \bar{B}\cdot\bar{ds} = 0$

Q1: b) $\because \bar{\nabla}\times\bar{\nabla}\times\bar{A} = \nabla(\nabla\cdot A) - \nabla^2 A$

$\therefore \nabla\times\nabla\times E = -\frac{\partial}{\partial t}\nabla\times B$ or

$\nabla(\nabla\cdot\frac{D}{\epsilon}) - \nabla^2 E = -\mu\frac{\partial}{\partial t}\nabla\times H \longrightarrow I$

$\because \nabla\cdot D = 0$ for source free medium

$\therefore -\nabla^2 E = -\mu\frac{\partial}{\partial t}\nabla\times H$

$\because \nabla\times H = \sigma E + \epsilon\frac{\partial E}{\partial t}$

$-\nabla^2 E = -\mu\sigma\frac{\partial E}{\partial t} + \mu\epsilon\frac{\partial^2 E}{\partial t^2}$

for harmonic fields:

$E = E_0 \exp(j\omega t)$

$\frac{\partial\bar{E}}{\partial t} = j\omega E_0 \exp(j\omega t) = j\omega\bar{E}$

$\therefore \frac{\partial^2 E}{\partial t^2} = -\omega^2 E$

②

$$\therefore \nabla^2 E = j\omega\mu\sigma E + (-\omega^2)\mu\epsilon E$$

$$= [j\omega\mu\sigma - \omega^2\mu\epsilon] E$$

$$\text{or } \nabla^2 E + [j\omega\mu\sigma + \omega^2\mu\epsilon] E = 0$$

$$\text{or } \nabla^2 E + \gamma^2 E = 0$$

$$\gamma^2 = j\omega\mu(\sigma - j\omega\epsilon)$$

$\bar{H}$ Can be obtained in the same way

$$\nabla^2 H + \gamma^2 H = 0$$

Q1:c

consider $\sigma = 0$

$$\therefore \nabla \times H = \epsilon \frac{\partial E}{\partial t} = j\omega\epsilon E$$

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{E_m}{\eta} e^{j(\omega t - \beta z)} & 0 & 0 \end{vmatrix} = j\omega\epsilon E$$

Hence find E

$\bar{S} = \bar{E} \times \bar{H} \Rightarrow$ the propagation direction

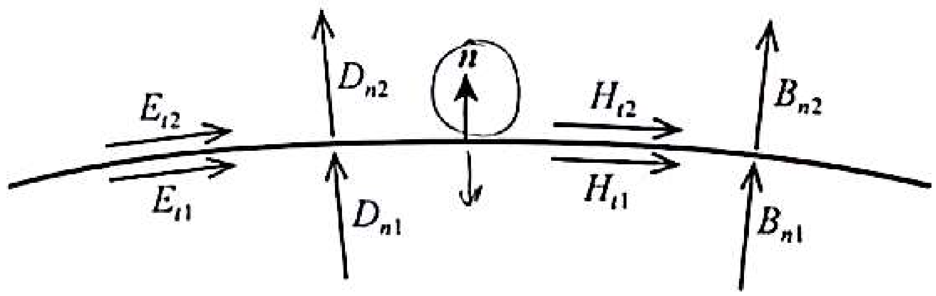
(Q2)a

Electromagnetic Field Boundary Conditions

Although the dynamic forms of Ampere's and Faraday's laws contain terms not found in the static forms of these equations [$\partial \boldsymbol{B}/\partial t$, $\partial \boldsymbol{D}/\partial t$], the boundary conditions developed for static fields are still valid for dynamic fields. The boundary conditions involving Ampere's law and Faraday's law are evaluated on rectangular paths that span the media interface. As the height of the rectangular path is allowed to approach zero, the contributions of the time-derivative terms vanish.

Note that the unit normal $\hat{\boldsymbol{n}}$ points into region 1.

Region 2 ($\mu_2, \epsilon_2, \sigma_2$)



Region 1 ($\mu_1, \epsilon_1, \sigma_1$)

$\hat{\boldsymbol{n}} \cdot (\boldsymbol{D}_1 - \boldsymbol{D}_2) = \rho_s$	$D_{n1} - D_{n2} = \rho_s$
$\hat{\boldsymbol{n}} \times (\boldsymbol{H}_1 - \boldsymbol{H}_2) = \boldsymbol{J}_s$	$H_{t1} - H_{t2} = J_s$
$\hat{\boldsymbol{n}} \cdot (\boldsymbol{B}_1 - \boldsymbol{B}_2) = 0$	$B_{n1} = B_{n2}$
$\hat{\boldsymbol{n}} \times (\boldsymbol{E}_1 - \boldsymbol{E}_2) = 0$	$E_{t1} = E_{t2}$

Q3(a)

good dielectric $\Rightarrow$ $\frac{\sigma}{\omega\epsilon} \ll 1$

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon)$$

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu \, j\omega\epsilon\left(1 + \frac{\sigma}{j\omega\epsilon}\right)}$$

$$= j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{j\omega\epsilon}\right)^{1/2}$$

$$\simeq j\omega\sqrt{\mu\epsilon}\left(1 + \frac{\sigma}{2j\omega\epsilon}\right)$$

$$\therefore \gamma \simeq \frac{\sigma}{2}\sqrt{\mu\epsilon} + j\omega\sqrt{\mu\epsilon}$$

$$= \alpha + j\beta$$

$$\alpha \simeq \frac{\sigma}{2}\sqrt{\mu\epsilon} \qquad \beta \simeq \omega\sqrt{\mu\epsilon}$$

$$\delta = \frac{1}{\alpha} = \frac{2}{\sigma\sqrt{\mu\epsilon}}$$

$$v_{ph} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$v_g = \frac{1}{\partial\beta/\partial\omega} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$E = E^+ \exp\left(-\frac{\sigma}{2}\sqrt{\mu\epsilon}\right)\exp(-j\omega\sqrt{\mu\epsilon})\exp(j\omega t)$$

$$+ E^- \exp\left(\frac{\sigma}{2}\sqrt{\mu\epsilon}\right)\exp(j\omega\sqrt{\mu\epsilon})\exp(j\omega t)$$

$$H = \frac{E}{Z}$$

3b

$\omega = 0.2\pi \times 10^{9}$

i $\frac{\sigma}{\omega\epsilon} = \frac{5\times10^{-6}}{0.2\times\pi\times10^{9}\times300\times\frac{1}{36\pi}\times10^{-9}}$

$= \frac{5\times10^{-6}\times36}{2\times300} = 0.3\times10^{-6} \ll 1$

$\therefore$ good dielectric (lossy dielectric)

ii) $\alpha, \beta, \delta, v_g, v_{ph}$ can be found

using values of (3a)

$$\eta = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma+j\omega\epsilon)}}$$

$$= j\omega\mu\left(j\omega\mu \cdot j\omega\epsilon\left(1+\frac{\sigma}{j\omega\epsilon}\right)\right)^{-1/2}$$

$$\simeq j\omega\mu \cdot (j\omega\sqrt{\mu\epsilon})^{-1}\left(1-\frac{\sigma}{2j\omega\epsilon}\right)$$

$$\simeq \sqrt{\frac{\mu}{\epsilon}}\left(1-\frac{\sigma}{2j\omega\epsilon}\right)$$

$$\simeq \sqrt{\frac{\mu}{\epsilon}} + j\frac{\sigma}{2\omega\epsilon}\sqrt{\frac{\mu}{\epsilon}}$$

$$H = \frac{E}{\eta}$$

4 a)

plarization: The direction of E-field.

Types: • Linear (horizontal, vertical)

• Circular

• elliptical.

Q4 b)

$$E(x,t) = 4\times10^{-3} e^{-6x} \cos(5\pi\times10^{9} - 10\pi x) a_y$$

$$+ E_0 e^{-\alpha x} \cos(\omega t - kx + \theta_0) a_z$$

i) $\alpha = 6$, $\beta = 10\pi$, $k = \beta$

sinc $\alpha \neq \beta$ $\therefore$ The medium is good dielectric

$\alpha = \frac{\omega}{2}\sqrt{\mu\epsilon}$ $\beta = \omega\sqrt{\mu\epsilon}$

$\omega = \frac{2\alpha}{\sqrt{\mu\epsilon}}$ $\beta = \omega\sqrt{\mu\epsilon}$

$\omega = \frac{2\alpha}{\sqrt{\mu_0\epsilon_0}\sqrt{\mu_r\epsilon_r}}$ $\beta = \omega\sqrt{\mu_0\epsilon_0}\cdot\sqrt{\mu_r\epsilon_r}$

ii)

1) $\tan^{-1}\frac{E_0}{4\times10^{-3}} = \tan 40^\circ$, $\theta_0 = 0^\circ$

2) $E_0 = 4\times10^{-3}$ $\theta_0 = 90^\circ$

3) $E_0 \neq 4\times10^{-3}$ $\theta_0 = 90^\circ$

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Q5a

B. Perpendicular Polarization

In this case, the E field is perpendicular to the plane of incidence (the xz-plane) as shown in Figure 10.17. This may also be viewed as the case where H field is parallel to the plane of incidence. The incident and reflected fields in medium 1 are given by

$$\mathbf{E}_{is} = E_{io} e^{-j\beta_1(x \sin\theta_i + z\cos\theta_i)}\, \mathbf{a}_y \tag{10.115a}$$

$$\mathbf{H}_{is} = \frac{E_{io}}{\eta_1}(-\cos\theta_i\, \mathbf{a}_x + \sin\theta_i\, \mathbf{a}_z)\, e^{-j\beta_1(x\sin\theta_i + z\cos\theta_i)} \tag{10.115b}$$

$$\mathbf{E}_{rs} = E_{ro} e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)}\, \mathbf{a}_y \tag{10.116a}$$

$$\mathbf{H}_{rs} = \frac{E_{ro}}{\eta_1}(\cos\theta_r\, \mathbf{a}_x + \sin\theta_r\, \mathbf{a}_z)\, e^{-j\beta_1(x\sin\theta_r - z\cos\theta_r)} \tag{10.116b}$$

while the transmitted fields in medium 2 are given by

$$\mathbf{E}_{ts} = E_{to} e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)}\, \mathbf{a}_y \tag{10.117a}$$

$$\mathbf{H}_{ts} = \frac{E_{to}}{\eta_2}(-\cos\theta_t\, \mathbf{a}_x + \sin\theta_t\, \mathbf{a}_z)\, e^{-j\beta_2(x\sin\theta_t + z\cos\theta_t)} \tag{10.117b}$$

Notice that in defining the field components in eqs. (10.115) to (10.117), Maxwell's equations (10.95) are always satisfied. Again, requiring that the tangential components of E and H be continuous at $z = 0$ and setting θ_r equal to θ_i, we get

$$E_{io} + E_{ro} = E_{to} \tag{10.118a}$$

$$\frac{1}{\eta_1}(E_{io} - E_{ro})\cos\theta_i = \frac{1}{\eta_2}E_{to}\cos\theta_t \tag{10.118b}$$

Expressing E_{ro} and E_{to} in terms of E_{io} leads to

$$\boxed{\Gamma_\perp = \frac{E_{ro}}{E_{io}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}} \tag{10.119a}$$

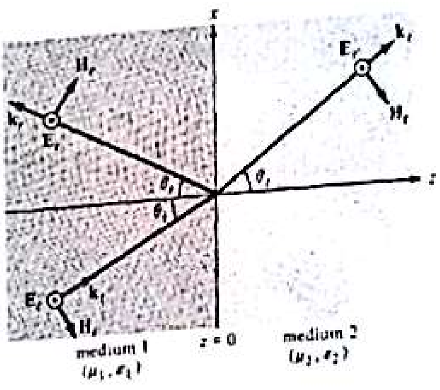

Figure 10.17 Oblique incidence with E perpendicular to the plane of incidence.

or

$$E_{ro} = \Gamma_\perp E_{io} \tag{10.119b}$$

and

$$\boxed{\tau_\perp = \frac{E_{to}}{E_{io}} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}} \tag{10.120a}$$

or

$$E_{to} = \tau_\perp E_{io} \tag{10.120b}$$

which are the *Fresnel's equations* for perpendicular polarization. From eqs. (10.119) and (10.120), it is easy to show that

$$1 + \Gamma_\perp = \tau_\perp \tag{10.121}$$

which is similar to eq. (10.83) for normal incidence. Also, when $\theta_i = \theta_t = 0$, eqs. (10.119) and (10.120) become eqs. (10.81) and (10.82) as they should.

For no reflection, $\Gamma_\perp = 0$ (or $E_r = 0$). This is the same as the case of total transmission ($\tau_\perp = 1$). By replacing θ_i with the corresponding Brewster angle $\theta_{B_\perp}$, we obtain

$$\eta_2 \cos\theta_{B_\perp} = \eta_1 \cos\theta_t$$

or

$$\eta_2^2 (1 - \sin^2\theta_{B_\perp}) = \eta_1^2 (1 - \sin^2\theta_t)$$

Incorporating eq. (10.104) yields

$$\sin^2\theta_{B_\perp} = \frac{1 - \mu_1\varepsilon_2/\mu_2\varepsilon_1}{1 - (\mu_1/\mu_2)^2} \tag{10.122}$$

Note that for nonmagnetic media ($\mu_1 = \mu_2 = \mu_o$), $\sin^2\theta_{B_\perp} \to \infty$ in eq. (10.122), so $\theta_{B_\perp}$ does not exist because the sine of an angle is never greater than unity. Also if $\mu_1 \neq \mu_2$ and $\varepsilon_1 = \varepsilon_2$, eq. (10.122) reduces to

$$\sin\theta_{B_\perp} = \sqrt{\frac{\mu_2}{\mu_1 + \mu_2}}$$

or

$$\tan\theta_{B_\perp} = \sqrt{\frac{\mu_2}{\mu_1}} \tag{10.123}$$

Although this situation is theoretically possible, it is rare in practice.